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ON GRANULAR COMPUTING VIA COVERING: NEW DEFINITION AND RELATED COVERING ROUGH SET

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Abstract

The Zoom-in and Zoom-out operators play an important role in the model of granular computing based on covering. In this paper, a new Zoom-in operator is defined, the combination operators formed by the Zoom-in and Zoom-out operators on the (granulated) universe of discourse are presented, and their relationships to covering rough set, topological space, and Galois connection are discussed. In particular, it is proved that a pair of approximation operators on the universe of discourse obtained by the combination of the Zoom-in operator and Zoom-out operator, are precisely the second type of covering-based lower and upper approximation operators.

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1. Introduction

In the real world, information is often granular and elements. It is within an information granule has to be dealt with as a whole rather than individually. The idea of information granularity has been explored in many fields, such as rough sets, fuzzy sets, cluster analysis, database, machine learning, and data mining [2, 4, 8, 15, 16, 19]. In recent years, there is a renewed interest in granular computing [1, 3, 5, 10, 14, 17, 18], and it has become increasingly important in information processing.

The model of granular computing can be regarded as a conceptual model or a mathematical model. Granular computing model is constructed by the concept of granular and the relevant operative symbol, and is used to reflect and describe the universe of discourse (i.e., real prototype) of various factors, forms, and quantitative relationships. The three major granular computing models are the words computing models [13], rough sets models [18], and the quotient space models [1]. Rough sets models maybe the most popular ones. In these models, many notions of granular computing can be defined and analyzed appropriately. In [14], Yao introduced a model of granular computing based on a partition of (or equivalent, a equivalent relation on) the universe of discourse. In [5], Ma generalized Yao's model from restricting equivalence relation to reflexive binary relation. On the other hand, She [12] extended Yao's model from partition to arbitrary covering on the universal of discourse.

This paper can be regarded as a further research on the covering model in [12]. We define a new Zoom-in operator and study its properties. As we will see that different combinations of Zoom-in and Zoom-out operators form different rough approximations on the universe of discourse and granulated universe of discourse, respectively. Specially, we prove that a pair of approximation operators (on the universe of discourse) obtained by the combination of the Zoom-in operator and

Zoom-out operator, are precisely the second type of covering based lower and upper approximation operators [7, 17, 18]. We also discuss the relationships between the operators stated above, topological space, and Galois connection. The content are arranged as follows. In Section 2, we recall some notions used in this paper and define a new Zoom-in operator. In Section 3, we study the combination operators formed by the Zoom-in and Zoom-out operators on the universe of discourse, and discuss relationships between these operators and covering rough set, topological space, and Galois connection. In Section 4, we research the combination operators formed by the Zoom-in and Zoom-out operators on the granulated universe of discourse, and discuss relationships between these operators and covering rough set, topological space, and Galois connection.

2. New Zoom-in Operator

In this section, we shall recall some notions, notations used in this paper and investigate a new Zoom-in operator.

Let U be a non-empty universe of discourse, C is a family of non-empty subsets of U. If $\bigcup C = U$, then C is called a covering of U. Let C be a finite covering on U. For each $x \in U$, the family

$$Md(x) = \{K \in C \mid x \in K, \forall S \in C, x \in S, S \subseteq K \Rightarrow S = K\},$$

is called the minimal description of x. C is called unary if for each $x \in U$, $|Md(x)| = 1$; C is called representative if for each $K \in C$, there exists a $x \in U$ such that $\forall S \in C, x \in S \Rightarrow K \subseteq S$. These definitions can be found in the literature [20].

Definition 2.1 ([13])**.** Let C be a finite covering on U. The mapping $\omega : 2^C \to 2^U$

$$\forall X \in 2^C, \quad \omega(X) = \{x \mid Md(x) \subseteq X\},$$

is called a *Zoom-in* operator.

Proposition 2.1 ([13]). *Let C be a finite covering on U. The Zoom-in operator has the following properties*:

(1) $\omega(\varnothing) = \varnothing$, $\omega(2^C) = U$.

(2) $\forall X, Y \in 2^C$, $\omega(X \cup Y) = \omega(X) \cup \omega(Y)$ *if and only if C is unary*.

(3) $\omega(X \cap Y) = \omega(X) \cap \omega(Y)$.

(4) $\omega(X)^c = \omega(X^c)$ *if and only if C is unary*.

(5) $X \subseteq Y \Leftrightarrow \omega(X) \subseteq \omega(Y)$ *if and only if C is representative*.

Definition 2.2 ([13]). Let C be a finite covering on U. Then the pair $(\overline{apr}, \underline{apr})$, where $\overline{apr}, \underline{apr} : 2^U \to 2^C$

$$\forall A \in 2^U, \overline{apr}(A) = \{X_i \in C \mid X_i \cap A \neq \varnothing\}, \underline{apr}(A) = \{X_j \in C \mid X_j \subseteq A\},$$

is called a *Zoom-out* operator.

In the following, we shall give a new Zoom-in operator based on covering and study its properties.

Definition 2.3. Let C be a finite covering on U. The mapping $\mu : 2^C \to 2^U$

$$\forall X \in 2^C, \quad \mu(X) = \bigcup\{K \mid K \in C, K \in X\},$$

is called a Zoom-in operator.

Proposition 2.2. *Let C be a finite covering on U. Then for each $X \in 2^C$, $\omega(X) \subseteq \mu(X)$.*

Proof. Let $x \in \omega(X)$, then there exists a $K \in C$ such that $x \in K \in Md(x)$. So $x \in \mu(X) = \bigcup\{K \mid K \in X\}$. By the arbitrariness of x, we get $\omega(X) \subseteq \mu(X)$.

The following example shows that the converse inclusion does not hold generally. Thus, the Zoom-in operator defined above is different from that in [12].

Example 2.1. Let $U = \{a, b, c, d\}$, $K_1 = \{a, b\}$, $K_2 = \{a, c\}$, $K_3 = \{b, d\}$, $C = \{K_1, K_2, K_3\}$. Then $\mu(\{K_1\}) = \{a, b\}$, $\mu(\{K_1, K_3\}) = \{a, b, d\}$, while $\omega(K_1) = \varnothing$, $\omega(\{K_1, K_3\}) = \{b, d\}$.

Remark 2.1. Let C is a partition on U. It is easily seen that $\mu(X) = \omega(X)$ for each $X \subseteq U$. Thus by [12], the operator μ can also be regarded as a generalization of the corresponding operator in [14]. Indeed, for each $x \in \mu(X) = \bigcup\{K \in C \mid K \in X\}$, then there exists $K \in X$ such that $x \in K$. Because C is a partition of U, thus $x \in Md(x) = K$, i.e., $x \in \omega(X)$. By the arbitrariness of x, we get $\mu(X) \subseteq \omega(X)$.

The next proposition lists some properties of the Zoom-in operator.

Proposition 2.3. *Let C be a finite covering on U. The Zoom-in operator $\mu : 2^C \to 2^U$ have the following properties*:

(1) $\mu(\varnothing) = \varnothing$; $\mu\left(2^C\right) = 2^U$.

(2) $X \subseteq Y \Rightarrow \mu(X) \subseteq \mu(Y)$.

(3) $\mu(X \cup Y) = \mu(X) \cup \mu(Y)$.

(4) $\mu(X)^c \subseteq \mu(X^c)$.

(5) *Let* $X \in C$, *then* $\mu(\{X\}) = X$.

Proof. Property (1) is obvious from the definition.

(2) For each $x \in \mu(X) = \bigcup\{K \mid K \in X\}$, there exists $K_x \in X$ such that $x \in K_x$. Since $X \subseteq Y$, then $x \in K_x \in Y$, i.e., $x \in \mu(Y)$.

(3) By the definition, we have

$$x \in \mu(X \cup Y) \Leftrightarrow x \in K \in X \cup Y \Leftrightarrow x \in K \in X \text{ or } x \in K \in Y$$

$$\Leftrightarrow x \in \mu(X) \cup \mu(Y).$$

(4) For all $x \in \mu(X)^c$, we have $x \notin K$ for each $K \in X$. Because C is a covering of U, thus there exists a $H \in C = X \cup X^c$ such that $x \in H \in X^c$, so $x \in \omega(X^c)$.

(5) Straightforward.

The following example show the reverse inequality of (4) does not hold in general.

Example 2.2. Let $U = \{a, b, c\}$, $K_1 = \{a, b\}$, $K_2 = \{a, c\}$, $C = \{K_1, K_2\}$. Taking $X = \{K_1\}$, then $X^c = \{K_2\}$, therefore $\mu(X)^c = \{c\} \neq \{a, c\} = \mu(X^c)$.

The property (3) in Proposition 2.1 does not hold as we show in the next example.

Example 2.3. Let $U = \{a, b, c\}$, $K_1 = \{a, b\}, K_2 = \{a, c\}$,$C = \{K_1, K_2\}$. Considering $X = \{K_1\}$, $Y = \{K_2\}$. Then $a \in \mu(X) \cap \mu(Y)$, but $a \notin \mu(X \cap Y)$ $= \mu(\varnothing)$.

3. The Approximation Operators on 2^U

In [7], the second type of covering-based rough sets is introduced. For each $X \subseteq U$, the covering lower and upper approximations are defined as follows:

$$X_* = \bigcup\{K \mid K \in C,\ K \subseteq X\};\ X^* = \bigcup\{K \mid K \in C,\ K \cap X \neq \varnothing\}.$$

The follow theorem shows that the lower and upper approximations are precisely the combinations of the Zoom-in and Zoom-out operators.

Proposition 3.1. *Let C be a finite covering on U and* $A \subseteq U$. *Then*

(1) $\mu \circ \overline{apr}(A) = \bigcup\{X \in C \mid X \cap A \neq \varnothing\} = X^*$.

(2) $\mu \circ \underline{apr}(A) = \bigcup\{X \in C \mid X \subseteq A\} = X_*$.

Proof. (1) $\mu \circ \overline{apr}(A) = \bigcup\{X \in C \mid X \in \overline{apr}(A)\} = \bigcup\{X \in C \mid X \cap A \neq \varnothing\}$.

(2) $\mu \circ \underline{apr}(A) = \bigcup\{X \in C \mid X \in \underline{apr}(A)\} = \bigcup\{X \in C \mid X \subseteq A\}$.

Besides, we can easily obtain the following corollary:

Corollary 3.1. *Let C be a finite covering on U and* $A \subseteq U$. *Then*

(1) $\mu \circ \overline{apr}(A) = \{x \in U \mid x \in K \in \overline{apr}(A)\} = \{x \in U \mid \exists X \in C, X \cap A \neq \varnothing, x \in X\}$.

(2) $\mu \circ \underline{apr}(A) = \{x \in U \mid x \in K \in \underline{apr}(A)\} = \{x \in U \mid \exists X \in C, x \in X \subseteq A\}$.

(3) *If* $A \in C$, *then* $\mu \circ \underline{apr}(A) = A$.

Corresponding to the properties of the second type of covering lower and upper approximations listed in the literature [21], we have the following results:

Proposition 3.2. *Let C be a finite covering on U. Then*

(1) $\mu \circ \overline{apr}(U) = U$, $\mu \circ \underline{apr}(U) = U$, $\mu \circ \overline{apr}(\varnothing) = \varnothing$, $\mu \circ \underline{apr}(\varnothing) = \varnothing$.

(2) $\mu \circ \underline{apr}(A) \subseteq A \subseteq \mu \circ \overline{apr}(A)$.

(3) $A \subseteq B \Rightarrow \mu \circ \underline{apr}(A) \subseteq \mu \circ \underline{apr}(B)$; $A \subseteq B \Rightarrow \mu \circ \overline{apr}(A) \subseteq \mu \circ \overline{apr}(B)$.

(4) $\mu \circ \overline{apr}(A \cup B) = \mu \circ \overline{apr}(A) \cup \mu \circ \overline{apr}(B)$.

(5) $\mu \circ \underline{apr}(A \cap B) = \mu \circ \underline{apr}(A) \cap \mu \circ \underline{apr}(B)$ *if and only if C is unary.*

(6) $(\mu \circ \underline{apr})(\mu \circ \underline{apr}(A)) = \mu \circ \underline{apr}(A)$.

It is well-known that there exists closed interrelationship between the theory of topologies and that of covering-based rough sets (resp., granular computing). Just before exhibiting this relation, we first give an interesting topological approached description for unary covering.

Recall that a non-empty set $\mathcal{B} \subseteq \mathcal{P}(X)$ is a base for a topology $\mathcal{J}$ on X if and only if $\bigcup\mathcal{B} = X$, and for each $B_1, B_2 \in \mathcal{B}$ and each $x \in B_1 \cap B_2$, there exists a $B_3 \in \mathcal{B}$ such that $x \in B_3 \subseteq B_1 \cap B_2$.

Proposition 3.3. *Let C be a covering on U, and C is unary if and only if C is a base for some topology on U.*

Proof. Let C be a unary covering. For all $K_1, K_2 \in C$, if $x \in K_1 \cap K_2$, then $x \in K_1 \cap K_2 = \bigcup_{y \in K_1 \cap K_2} Md(y)$. Thus, there exists a $y \in K_1 \cap K_2$ such that $x \in Md(y) \subseteq K_1 \cap K_2$. That means, C is a base for some topology on U.

On the other hand, let C be a base for some topology on U. For each $x \in U$, taking $K_1, K_2 \in Md(x)$. To prove that C is unary, it suffices to check that $K_1 = K_2$. Indeed, by $x \in K_1 \cap K_2$, we have a $K \in C$ such that $x \in K \subseteq K_1 \cap K_2$. By the definition of $Md(x)$, we obtain that $K = K_1 = K_2$ as desired.

Definition 3.1. Let U be a non-empty universal of discourse. Then the mapping $i : 2^U \to 2^U$ satisfying the following conditions: $\forall A, B \subseteq U$,

(1) $i(U) = U$.

(2) $i(A \cap B) = i(A) \cap i(B)$.

(3) $i(A) \subseteq A$,

is called an interior operator on U. In addition, i is called a topological interior operator on U if it further satisfies:

(4) $i(i(A)) = i(A)$.

Dually, one can define the so called (topological) closure operator.

Remark 3.1. By Proposition 3.3, we observe easily that the operator $u \circ \overline{apr}$ is a closure operator on U. If C is unary, then the operator $\mu \circ \underline{apr}$ is a topological interior operator on U.

Definition 3.2 ([13])**.** Let U be a non-empty universal of discourse. Then the pair (f, g), where $f, g : 2^U \to 2^U$, is said to be a *Galois connection* on U, if it satisfies the following rules:

(1) $A_1 \subseteq A_2 \Rightarrow f(A_1) \subseteq f(A_2),\ g(A_1) \subseteq g(A_2)$,

(2) $f(g(A)) \subseteq A,\ g(f(A)) \supseteq A$.

Theorem 3.1. *Let U be a non-empty universe of discourse, C be a unary covering on U. Then the pair $\left(\mu \circ \overline{apr}, \mu \circ \underline{apr}\right)$ is a Galois connection on U if and only if C is a partition on U.*

Proof. *Necessity:* To prove C is a partition on U, we need to check

$$\forall X_i, X_j \in C, \quad X_i \cap X_j \neq \varnothing \Rightarrow X_i = X_j.$$

In fact, taking $x \in X_i \cap X_j$ and assuming $Md(x) = \{X_k\}$. Then by the definition of $Md(x)$, we have $X_k \subseteq X_i$ and $X_k \subseteq X_j$. Because $\left(\mu \circ \overline{apr}, \mu \circ \underline{apr}\right)$ is a Galois connection, thus $\mu \circ \overline{apr}\left(\mu \circ \underline{apr}(X_k)\right) \subseteq X_k$. From Proposition 3.1 and Corollary 3.1 (3), we have

$$\mu \circ \overline{apr}\left(\mu \circ \underline{apr}(X_k)\right) = \bigcup \{X \in C \mid X \cap X_k \neq \varnothing\}.$$

By $x \in X_k \cap X_i \cap X_j$, we have

$$X_i, X_j \subseteq \mu \circ \overline{apr}\left(\mu \circ \underline{apr}(X_k)\right) \subseteq X_k.$$

Thus $X_i = X_j = X_k$.

Sufficiency: Let C is a partition on U. Then $\mu = \omega$ by Remark 2.1. Thus the sufficiency has been proved by Proposition 7 in [14].

4. The Approximation Operators on 2^C

In this section, we examine the relationships of a topological spaces and different combination operators formed by the Zoom-in and Zoom-out operators. Furthermore, we study the dual Galois connections formed by these combination operators.

Proposition 4.1. *Let C be a finite covering on U and $X \subseteq 2^C$. Then*

(1) $\overline{apr} \circ \mu(X) = \{S \in C \mid \exists K_i \in X, S \cap K_i \neq \varnothing\}$.

(2) $\underline{apr} \circ \mu(X) = \{S \in C \mid S \subseteq \cup K_i, K_i \in X\}$.

Proof. By the definition, we have

$$\overline{apr} \circ \mu(X) = \{S \in C \mid S \cap \mu(X) \neq \varnothing\} = \{S \in C \mid \exists K_i \in X, S \cap K_i \neq \varnothing\}.$$

$$\underline{apr} \circ \mu(X) = \{S \in C \mid S \subseteq \mu(X)\} = \{S \in C \mid S \subseteq \cup K_i, K_i \in X\}.$$

The following proposition lists the properties of the operators $\mu \circ \overline{apr}$, $\mu \circ \underline{apr}$.

Proposition 4.2. *Let C be a finite covering on U. For any $X, Y \in 2^C$,*

(1) $\overline{apr} \circ \mu(2^C) = 2^C$, $\underline{apr} \circ \mu(2^C) = 2^C$, $\overline{apr} \circ \mu(\varnothing) = \varnothing$, $\underline{apr} \circ \mu(\varnothing) = \varnothing$.

(2) $X \subseteq \underline{apr} \circ \mu(X)$, $\overline{apr} \circ \mu(X)$.

(3) $X \subseteq Y \Rightarrow \underline{apr} \circ \mu(X) \subseteq \underline{apr} \circ \mu(Y)$, $\overline{apr} \circ \mu(X) \subseteq \overline{apr} \circ \mu(Y)$.

(4) $\overline{apr} \circ \mu(X \cup Y) = \overline{apr} \circ \mu(X) \cup \overline{apr} \circ \mu(Y)$.

(5) $(\underline{apr} \circ \mu)(\underline{apr} \circ \mu(X)) = \underline{apr} \circ \mu(X)$.

Proof. (1)-(3) are straightforward.

(4) By the definition, we have

$$S \in \overline{apr} \circ \mu(X \cup Y) \Leftrightarrow \exists K_i \in X \cup Y,\ S \cap K_i \neq \varnothing$$

$$\Leftrightarrow \exists K_i \in X,\ S \cap K_i \neq \varnothing \text{ or } \exists K_i \in Y,\ S \cap K_i \neq \varnothing$$

$$\Leftrightarrow S \in \overline{apr} \circ \mu(X) \text{ or } S \in \overline{apr} \circ \mu(Y)$$

$$\Leftrightarrow S \in \overline{apr} \circ \mu(X) \cup \overline{apr} \circ \mu(Y).$$

(5) $\forall X \in 2^C$,

$$(\underline{apr} \circ \mu)(\underline{apr} \circ \mu(X)) = \{S \in C \mid S \subseteq \cup K,\ K \in \underline{apr} \circ \mu(X)\}$$

$$= \{S \in C \mid S \subseteq \cup K,\ K \subseteq \cup M,\ M \in X\}$$

$$= \{S \in C \mid S \subseteq \cup M,\ M \in X\}$$

$$= \underline{apr} \circ \mu(X).$$

Remark 4.1. It is proved in [14] that the Zoom-in operator ω possess the property (4) only when C being a unary covering. In addition, it is easily seen that the operator $\overline{apr} \circ \mu$ indeed preserve the arbitrary unions.

The next example shows that the multiplication and idempotency of the operator $\underline{apr} \circ \mu$ are no longer valid.

Example 4.1. Let $U = \{a, b, c, d\}$, $K_1 = \{a, b\}, K_2 = \{a, c\}, K_3 = \{b, d\}$, $C = \{K_1, K_2, K_3\}$.

Letting $X = \{K_1\}$, $Y = \{K_2, K_3\}$, then $K_1 \in \underline{apr} \circ \mu(X) \cap \underline{apr} \circ \mu(Y)$. But $\underline{apr} \circ \mu(X \cap Y) = \varnothing$. Thus $\underline{apr} \circ \mu(X) \cap \underline{apr} \circ \mu(Y) \neq \underline{apr} \circ \mu(X \cap Y)$.

Taking $X = \{K_3\}$. It is easy to check that $K_2 \in \overline{apr} \circ \mu(\overline{apr} \circ \mu(X))$ but $K_2 \notin \overline{apr} \circ \mu(X) = \varnothing$.

Definition 4.1. Let U be a non-empty domain of discourse.

(1) A function $i : 2^C \to 2^C$ it is called a pretopological interior operator on 2^C if for each $X, Y \in 2^C$:

(I) $i(2^C) = 2^C$.

(II) $X \subseteq Y \Rightarrow i(X) \subseteq i(Y)$.

(III) $i(i(X)) = i(X)$.

(2) A function $cl : 2^C \to 2^C$ it is called a pretopological closure operator on 2^C if for each $X, Y \in 2^C$:

(I) $cl(\varnothing) = \varnothing$.

(II) $cl(X \cup Y) = cl(X) \cup cl(Y)$.

(III) $X \subseteq cl(X)$.

Remark 4.2. By Proposition 4.2, we observe easily that the operator $\underline{apr} \circ \mu$ (resp., $\overline{apr} \circ \mu$) is a pretopological interior (resp., closure) operator on 2^C.

The following proposition exhibits us the relationship between the operators $\overline{apr} \circ \mu$, $\underline{apr} \circ \mu$ and Galois connection.

Theorem 4.1. *Let U be a non-empty universe of discourse, C be a unary covering on U. Then the pair $(\overline{apr} \circ \mu, \underline{apr} \circ \mu)$ is a Galois connection if and only if C is a partition on U.*

Proof. *Necessity*: To prove C is a partition on U, we need to check

$$\forall X_i, X_j \in C, \quad X_i \cap X_j \neq \varnothing \Rightarrow X_i = X_j.$$

In fact, taking $x \in X_i \cap X_j$ and assuming $Md(x) = \{X_k\}$. Then by the definition of $Md(x)$, we have $X_k \subseteq X_i$ and $X_k \subseteq X_j$. Because $\left(\overline{apr} \circ \mu,\ \underline{apr} \circ \mu\right)$ is a Galois connection, thus

$$\overline{apr} \circ \mu\left(\underline{apr} \circ \mu(\{X_k\})\right) \subseteq \{X_k\}.$$

By Proposition 2.3 (5) and Proposition 4.2, we have $\underline{apr} \circ \mu(\{X_k\}) = \underline{apr}(X_k)$. So,

$$\begin{aligned}\overline{apr} \circ \mu\left(\underline{apr} \circ \mu(\{X_k\})\right) &= \left\{S \in C \,\middle|\, \exists K_i \in \underline{apr}(X_k),\ S \cap K_i \neq \varnothing\right\} \\ &= \{S \in C \mid \exists K_i \in C \text{ and } K_i \subseteq X_k,\ S \cap K_i \neq \varnothing\} \\ &= \{S \in C \mid S \cap K_k \neq \varnothing\} \subseteq \{X_k\}.\end{aligned}$$

Because $x \in X_k \cap X_i \cap X_j$, thus

$$X_i,\ X_j \in \overline{apr} \circ \mu\left(\underline{apr} \circ \mu(\{X_k\})\right) \subseteq \{X_k\}.$$

So, $X_i = X_j = X_k$ as desired.

Similar to Theorem 3.1, the sufficiency has been proved by Proposition 12 in [14].

5. Conclusion

We define a new Zoom-in operator and consider the combinations of Zoom-in and Zoom-out operators [12]. It is proved that the combination of Zoom-in operator with Zoom-out operator (resp., Zoom-out operator with Zoom-in operator) form a pair of approximation operators on the (resp., granulated) universe of discourse. In particular, it is shown that the approximation operators on the universe of discourse are precisely the second type of covering-based approximation operators. In addition, we establishes the interrelationship between these approximation operators, topological spaces, and Galois connections.

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